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L-5001
Reg. No. : $\qquad$
Name : $\qquad$

Fifth Semester B.Tech. Degree Examination, March 2021.
13.501 : ENGINEERING MATHEMATICS - IV (BCHMPSU)

## (2013 Scheme)

Time : 3 Hours
Max. Marks : 100

## PART - A

Answer all questions. Each question carries 4 marks.

1. A die is tossed thrice. Getting 5 or 6 is considered as a success. If $X$ denotes probability of success, find $P(X=1)$.
2. The pdf of a random variable $X$ is $f(x)=c e^{-|x|},-\infty<x<\infty$. Find $c$.
3. The average marks in Mathematics of a sample of 100 students was 51 with a S.D. of 6 marks. Find the test statistics to check whether this have been a random sample from a population having average 50?
4. Explain slack and surplus variable using suitable examples.
5. Write the dual of the following LPP.

Maximise $z=3 x_{1}+x_{2}+2 x_{3}$,
Subject to $x_{1}+x_{2}+x_{3} \leq 5$

$$
\begin{aligned}
& 2 x_{1}+x_{3} \leq 10, \\
& x_{2}+3 x_{3} \leq 15, \\
& x_{1}, x_{2}, x_{3} \geq 0 .
\end{aligned}
$$

## PART - B

Answer any one questions from each Module. Each question carries $\mathbf{2 0}$ marks.

## Module - I

6. (a) The mileage which a car owner gets with a certain kind of tyre is a random variable having an exponential distribution with mean $60,000 \mathrm{~km}$. Find the probabilities that one of these tyres will last.
(i) at least $55,000 \mathrm{~km}$
(ii) atmost $65,000 \mathrm{~km}$
(b) The probability of an item produced by a certain machine will be defective is 0.05 . If the produced items are sent to the market in packets of 20 , find the number of packets containing
(i) atleast 2
(ii) exactly 2
(iii) atmost 2 defective items in a consignment of 1000 packets using Poisson distribution.
(c) Derive the mean and variance of uniform distribution.
7. (a) A random variable $X$ has the following probability density function.

$$
\begin{array}{ccccccccc}
x: & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
f(x): & 0 & k & 2 k & 2 k & 3 k & k^{2} & 2 k^{2} & 7 k^{2}+k
\end{array}
$$

Find
(i) $k$
(ii) $P(x<6), P(x \geq 6), P(3<x \leq 6)$
(iii) Find minimum value of $c$ so that $P(X \leq c)>\frac{1}{2}$
(b) In an examination 30\% of the candidates obtained marks below 40 and $10 \%$ of the candidates got above 75 marks. Assuming that the marks are normally distributed, find the mean and standard deviation of the distribution.

## Module - II

8. (a) Apply the method of least square to, fit a straight line $y=a+b x$ in the following data: $(x, y):(1,0),(2,1),(3,1),(4,2)$.
(b) The average income of persons was Rs. 210 with S.D. of Rs. 10 in sample of 100 people of a city. For another sample of 150 persons, the average income was Rs.220, With S.D. of Rs.12. The S.D. of the incomes of the people of the city was Rs.11. Test whether there is any significant difference between the average incomes of the localities.

9. (a) Calculate the Karl Pearson's coefficient of correlation between the marks in Economics and statistics for 10 students form the following data:

| Roll No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Marks in Economics | 78 | 36 | 98 | 25 | 75 | 82 | 90 | 62 | 65 | 39 |
| Marks in Statistics | 84 | 51 | 91 | 60 | 68 | 62 | 86 | 58 | 53 | 47 |

(b) A machine produced 16 defective articles in a batch of 500 . After servicing it produced 3 defectives in a batch of 100. Has the machine improved?
10. (a) Use simplex method of

Maximize $z=2 x_{1}+5 x_{2}$
Subject to $x_{1}+4 x_{2} \leq 24$,

$$
\begin{aligned}
& 3 x_{1}+x_{2} \leq 21, \\
& x_{1}+x_{2} \leq 9 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

(b) Solve using Big $M$ method:

Minimise $z=4 x_{1}+3 x_{2}$,
Subject to $2 x_{1}+x_{2} \geq 10$

$$
\begin{aligned}
& -3 x_{1}+2 x_{2} \leq 6 \\
& x_{1}+x_{2} \geq 6 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

OR
11. (a) Use simplex method to

Minimise $z=x_{1}-3 x_{2}+2 x_{3}$,
Subject to $3 x_{1}-x_{2}+2 x_{3} \leq 7$,

$$
\begin{aligned}
& -2 x_{1}+4 x_{2} \leq 12 \\
& -4 x_{1}+3 x_{2}+8 x_{3} \leq 10 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{aligned}
$$

(b) Solve using big $M$ method:

Maximise $z=2 x_{1}+3 x_{2}$,
Subjec to $x_{1}+2 x_{2} \leq 4$,

$$
\begin{aligned}
& x_{1}+x_{2}=3 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$



## Module - IV

12. (a) Apply the principle of duality to

Minimize $z=4 x_{1}+3 x_{2}+6 x_{3}$
Subject to $x_{1}+x_{3} \geq 2$,

$$
\begin{aligned}
& x_{2}+x_{3} \geq 5 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{aligned}
$$

(b) Find the initial feasible solution to the transportation problem.

|  | D | E | F | G | Available |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 11 | 13 | 17 | 14 | 250 |
| B | 16 | 18 | 14 | 10 | 300 |
| C | 21 | 24 | 13 | 10 | 400 |
| Requirement | 200 | 225 | 275 | 250 |  |

OR
13. (a) Apply the principle of duality to

Minimize $z=2 x_{1}+x_{2}$,
Subject to $3 x_{1}+x_{2} \geq 3$

$$
\begin{aligned}
& 4 x_{1}+3 x_{2} \geq 6 \\
& x_{1}+2 x_{2} \geq 3 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

(b) Solve the assignment problem

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| A | 10 | 12 | 19 | 11 |
| B | 5 | 10 | 7 | 8 |
| C | 12 | 14 | 13 | 11 |
| D | 8 | 15 | 11 | 9 |


( $4 \times 20=80$ Marks)

