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L – 5003

Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, March 2021.

13.501 : ENGINEERING MATHEMATICS – IV (AFRT) (COMPLEX ANALYSIS AND LINEAR ALGEBRA)

(2013 Scheme)

Time : 3 Hours

Max. Marks : 100

PART – A

Answer **all** questions. Each question carries **4** marks.

1. Find the constants a and b for which the function $f(z)=(x+ay)+i(2x+by)$ is analytic.

2. Find the image of the real axis $y=0$ in the w-plane under the transformation $w=z^2$.

3. Check whether the vectors $\{(1,-1,0),(1,3,-1),(5,3,-2)\}$ are linearly independent in $\mathbb{R}^3$.

4. Evaluate $\int_C \frac{e^z \sin z}{z+2}\, dz$ where C is the circle $|z|=1$.

5. Find k so that the vectors $(2,-2,-3,k)$ and $(k,3,2,2)$ are orthogonal in $\mathbb{R}^4$.

(5 × 4 = 20 Marks)

P.T.O.

PART – B

Answer **any one full** questions from **each** Module. Each question carries **20** marks.

Module – I

6. (a) Show that $f(z) = \cos z$ is analytic every where and hence find its derivative. **7**

   (b) Show that $u = x^2 - y^2 - y$ is harmonic and then find the corresponding harmonic conjugate and the analytic function. **7**

   (c) Find the image of $|z - 3i| = 3$ under the transformation $w = \dfrac{1}{z}$. **6**

OR

7. (a) Check whether $f(z) = \begin{cases} \dfrac{\text{Re}^z}{1-|z|} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$ is continuous at $z = 0$. **7**

   (b) Find the analytic function whose real part is $x^3 - 3xy^2$. Also find the imaginary part. **7**

   (c) Find the image of the semi-infinite strip $x > 0$ and $0 < y < 2$, under the transformation $w = 1^0 z + 1$. **6**

Module – II

8. (a) Evaluate $\int_C \dfrac{e^z dz}{(z-1)(z^2+4)}$ where C is $|z| = 1.5$. **7**

   (b) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{\sqrt{2} - \cos\theta}$. **7**

   (c) Obtain the Laurent's series expansion of $\dfrac{1}{(z-2)(z-3)}$ in power of $(z-2)$. **6**

OR

9. (a) Evaluate $\int_C \frac{zdz}{(z-2)(z-4)}$ using Cauchy's Residue Theorem, where C is a circle **7**

(i) $|z|=1$

(ii) $|z|=3$

(b) Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$. **7**

(c) Expand $f(z)=\frac{\cos z}{z-\pi}$ about $z=\pi$ in Taylor's series. **6**

Module – III

10. (a) Find a basis and dimensions for the row space, column space and null space of the matrix $\begin{bmatrix} 1 & 2 & 0 & 2 & 5 \\ -2 & -5 & 1 & -1 & -8 \\ 0 & -3 & 3 & 4 & 1 \\ 3 & 6 & 0 & -7 & 2 \end{bmatrix}$. **10**

(b) Check whether $v=(2,-5,3)\in\mathbb{R}^3$ can be expressed as a linear combination of vectors $v_1=(1,-3,2)$, $v_2=(2,-4,-1)$ and $v_3=(1,-5,7)$ **10**

OR

11. (a) Let $A=\begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $w=\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Determine whether W is in $C(A)$ and $N(A)$. **10**

(b) Let $V=\mathbb{R}^3$, $S=\{(x,y,z): 3x-2y+z=0, 4x+5y=0\}$. Find the dimension and basis of S. **10**

Module – IV

12\. (a) Find the least square of $Ax = b$ for $a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Also find the least square error. **10**

(b) Find all vectors in $\mathbb{R}^3$, that are orthogonal to $(1,1,1)$ and $(1,-1,0)$. Produce an orthonormal basis from these vectors. **10**

OR

13\. Use Gram-Schmidt method to find an orthonormal basis for $\mathbb{R}^4$ from the basis $\{(1,1,1,1),(1,1,2,4),(1,2,-4,-3)\}$. **20**

(4 × 20 = 80 Marks)