(Pages : 4) **L – 5002**

Reg. No. : ....

Name : ....

Fifth Semester B.Tech. Degree Examination, March 2021

13.501 : ENGINEERING MATHEMATICS – IV (E)
(PROBABILITY, RANDOM PROCESSES AND NUMERICAL TECHNIQUES)

(2013 Scheme)

Time : 3 Hours Max. Marks : 100

PART – A

(Answer **all** questions. Each question carries **4** marks each.)

1. Evaluate $\int_1^0 \frac{x}{\sin x}\,dx$ using Trapezoidal Rule with $h = 0.2$

2. Find an approximate solution to $x^4 - x - 10 = 0$ using Newton-Raphson method correct to three decimals.

3. A continuous random variable X has a pdf $f(x) = 3x^2;\ 0 \le x \le 1$. Find the value of '$a$' such that $p\{x \le a\} = p\{x > a\}$

4. Let X be a normal random variable with $\mu = 3$ and $\sigma^2 = 9$. Find $P\{x > 0\}$ and $P\{\,|X - 3| > 6\,\}$.

5. Let A and B be independent random variables with $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = K$. Show that the random process $X(t) = A\cos wt + B\sin wt$ is wide sense stationary.

(5 × 4 = 20 Marks)

P.T.O.

PART – B

Answer **ONE** full question from each module.

Each question carries **20** marks.

Module I

6. (a) Find an approximate solution to $\tan x + \tanh x = 0$ correct to three decimals using Bisection method.

   (b) Find the cubic polynomial which takes the values: $y(1) = 24$, $y(3) = 120$, $y(5) = 336$ and $y(7) = 720$. Obtain the value of $y(8)$.

   (c) Using Gauss-Seidel iteration solve:

   $4x + y + 2z = 4$

   $3x + 5y + z = 7$

   $x + y + 3z = 3$

OR

7. (a) Using Regula-falsi method, find a real root correct to three decimals of $x^3 + x^2 + x + 7 = 0$.

   (b) Solve $2x + y - z = -1$

   $x - 2y + 3z = 9$

   $3x - y + 5z = 14$

   using Gauss Elimination method.

   (c) Find the value of $y(2)$ from the following data using Lagrange's interpolation.

x	0	1	3	4	5
y	0	1	81	256	625

Module II

8. (a) Using the following table, find the area bounded by the curve and the x-axis from $x = 7.47$ to $x = 7.52$.

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

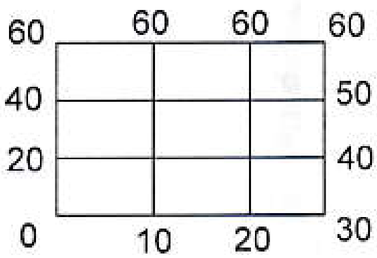
(b) For the differential equation $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$ with $y(0) = 1$ and $y'(0) = 0$. Use Taylor's series method for finding the value of y(0.1).

(c) For the differential equation $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, compute $y(0.2)$ and $y(0.3)$ using modified Euler's method.

OR

9. (a) Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} = 1 + y^2$; $y(0) = 0$ at $x = 0.6$ with $h = 0.2$.

(b) The function $u(x, y)$ satisfies Laplace equation at all points within the squares given below and has the boundary values as indicated. Compute a solution correct to two decimal places using finite difference method.



Module III

10. (a) Find the mean and variance of a Normal distribution.

(b) If a random variable X has a Poisson distribution with $P(1) = P(2)$. Find $P\{X=4\}$.

(c) A point is chosen at random on the line segment [0,2]. What is the probability that the point chosen lies between 1 and 1.5.

OR

11. (a) Fit a Poisson distribution for the following data:

x	0	1	2	3	4
Frequency	192	100	24	3	1

(b) A continuous random variables X has the probability density function

$$f(x) = \begin{cases} Kxe^{-\lambda x}; \ x \geq 0, \ \lambda > 0 \\ 0 \quad \text{otherwise} \end{cases}$$

(i) Find the value of K.

(ii) Find mean and variance of X.

(c) A irregular 6 face die is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 1000 sets.

Module IV

12. (a) The joint probability distribution of X and Y is given by

Y \ X	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	0
2	0	$\frac{1}{9}$	$\frac{1}{5}$
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$

(i) Find the marginal distribution of X and Y

(ii) Find the conditional distribution of Y given $X = 2$

(iii) Find $P\{X \leq 2, Y = 3\}$

(b) Consider the random process $X(t) = \cos(wt + \theta)$ where θ is uniformly distributed in $[-\pi, \pi]$ Check whether $X(t)$ is stationary.

(c) Let $\{X_n : n \geq 0\}$ is a sequence of random variable with mean 0 and variance 1. Prove that $\{X_n : n \geq 0\}$ is a wide-sense stationary.

OR

13. (a) Let $\{X(t)\}$ be a WSS process with mean 5 and autocorrelation $25 + 4e^{-2|\tau|}$ Find the autocorrelation and spectrum of $Y(t) = 2X(t) + 3x'(t)$

(b) Define a Poisson process. Check whether Poisson process is stationary. Find the autocorrelation, autocovariance and correlation coefficient of a Poisson process. **(4 × 20 = 80 Marks)**